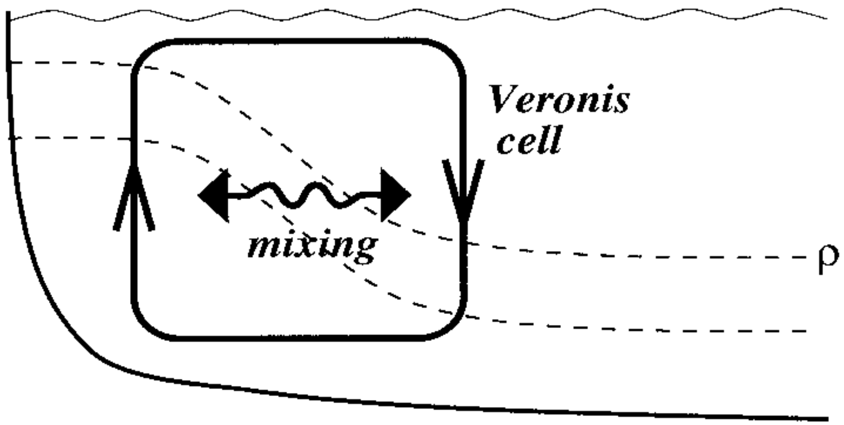
Brief tutorial on Gent and McWilliams and related schemes as relevant to ICOM

- why do we need GM?
- what is GM?
- implementation
- residual-mean formulation
- do we need GM in eddy-permitting calculations?

Problem with lateral diffusion of temperature and salinity (explicit or implicit):



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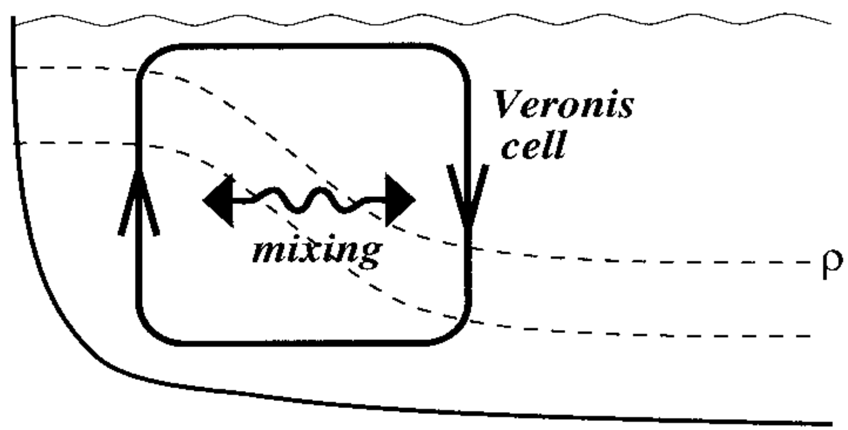
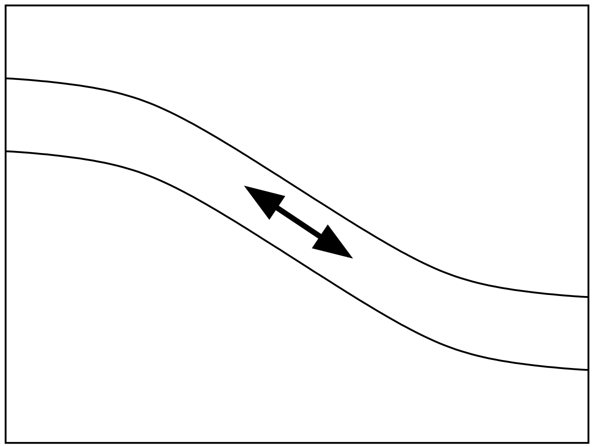


Removal of spurious lateral diffusion (Danabasoglu et al. 1994, Griffies et al. 1998, ...):

- sharper pycnocline;
- removal of premature upwelling of NADW within Gulf Stream;
- more realistic heat transports;
- partial removal of Deacon cell in Southern Ocean;
- confinement of convection to regions in which it is known to occur;
- improved estimates of carbon uptake and other tracer fluxes.

• orientate diffusion along isopycnals (Redi 1982)

cannot be a complete eddy parameterisation - no effect on density!



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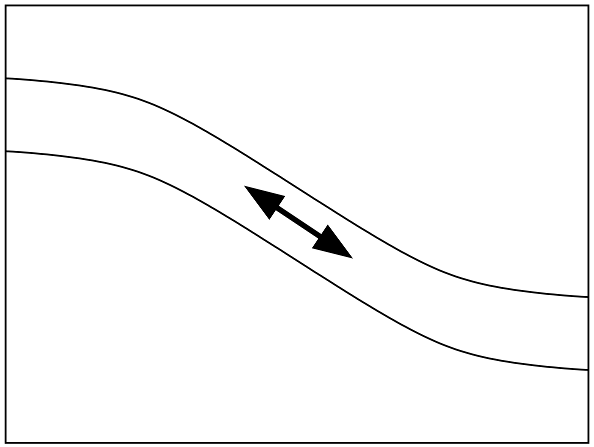
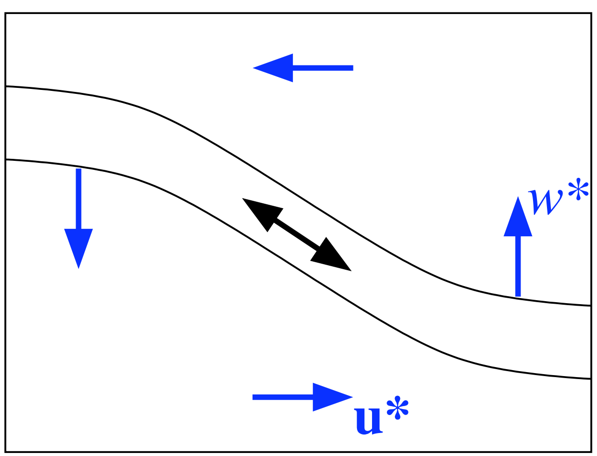


• add advection by *eddy transport velocity* (bolus velocity) - acts to flatten isopycnals (Gent and McWilliams 1990, Gent et al. 1995)



$$\mathbf{u}^* = \frac{\partial}{\partial z}\left(\kappa \frac{\boldsymbol{\nabla} b}{\partial b/\partial z}\right), \quad w^* = -\boldsymbol{\nabla}\cdot\left(\kappa \frac{\boldsymbol{\nabla} b}{\partial b/\partial z}\right),$$

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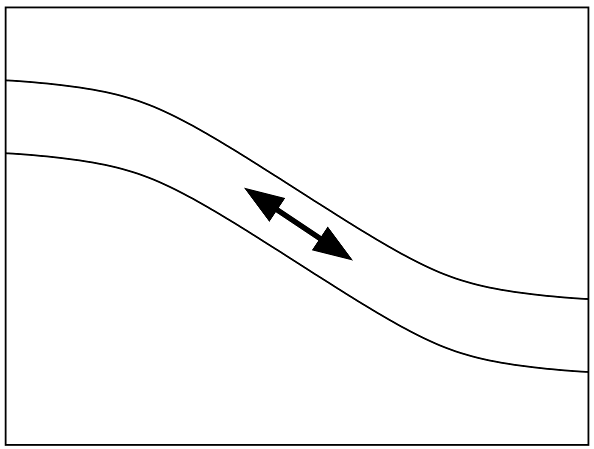

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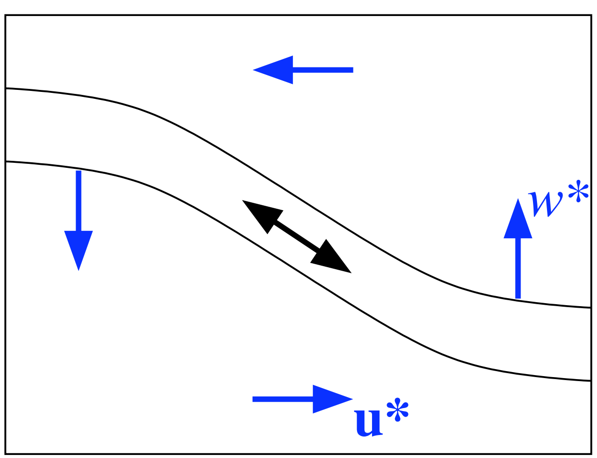

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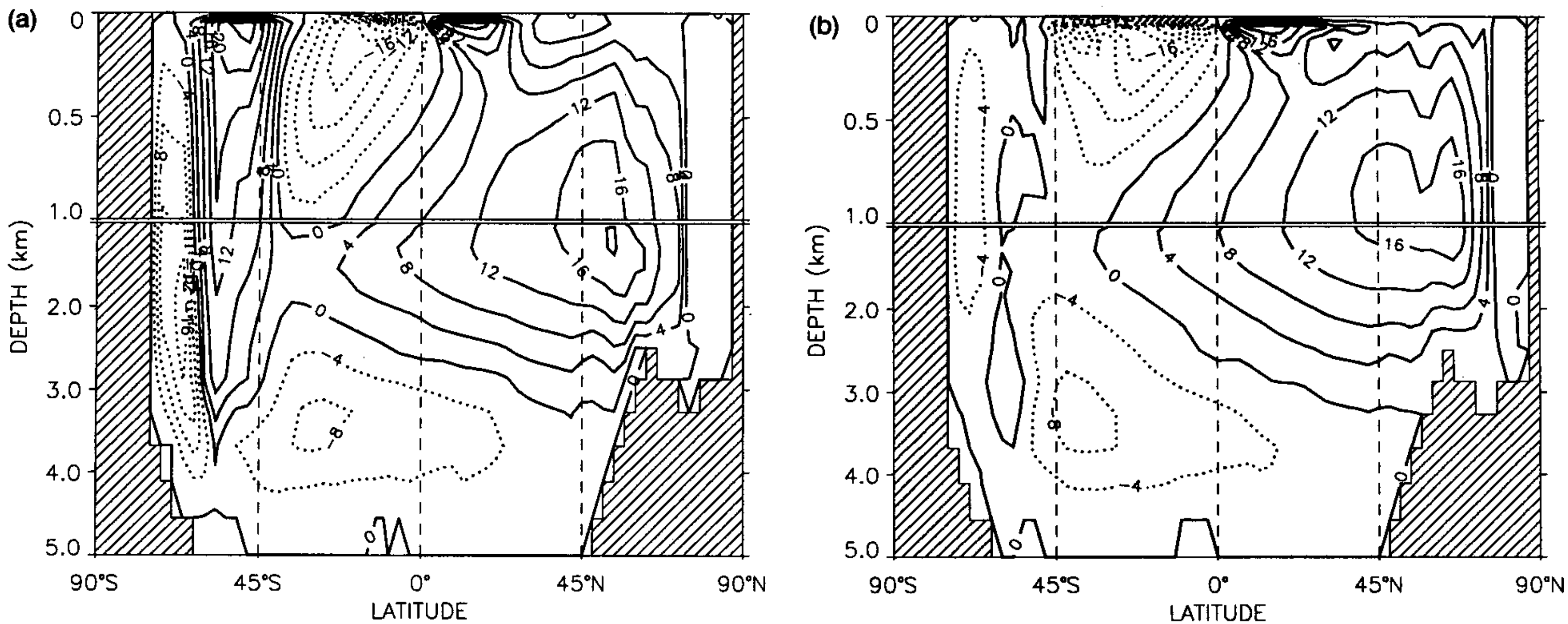
equivalent to lateral diffusion of isopycnal elevation



$\overline{\mathbf{u}}$

$\overline{\mathbf{u}} + \mathbf{u}^*$

(Danabasoglu and McWilliams 1995)

Implementation:

- small-slope approximation;
- can write tracer advection by **u*** as a skew-symmetric diffusion;

  gives total tracer flux:

$$\begin{pmatrix} F^{(x)} \\ F^{(y)} \\ F^{(z)} \end{pmatrix} = \begin{pmatrix} A_I & 0 & (A_I - \kappa)\, S_x \\ 0 & A_I & (A_I - \kappa)\, S_y \\ (A_I + \kappa)\, S_x & (A_I + \kappa)\, S_y & A_I\, S^2 \end{pmatrix} \begin{pmatrix} C_{,x} \\ C_{,y} \\ C_{,z} \end{pmatrix}$$

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Large literature on choice of k:

$$\kappa = L^2/T$$

- e.g., L = width of "baroclinic zone" $T^{-1} = \frac{f}{D}\int_{-D_b}^{-D_t} Ri^{-1/2}\, dz$ (Visbeck et al. 1997);

- apply linear baroclinic instability theory (Killworth 1997);
- ...

Residual-mean formulation

Alternatively, can transform equations such that all eddy buoyancy fluxes appear in the momentum equations (e.g., Ferreira and Marshall 2006):

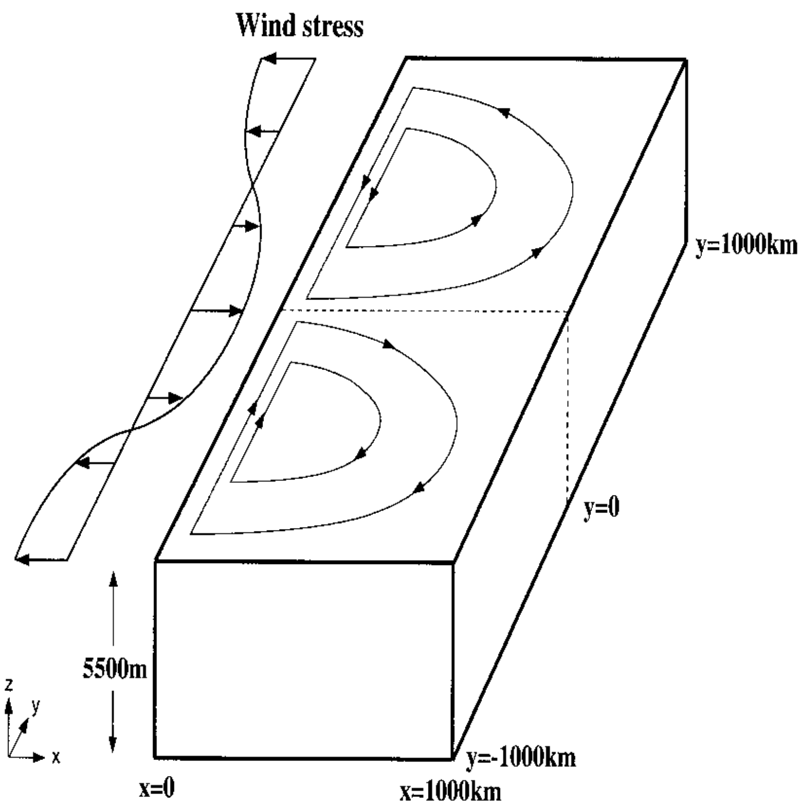
$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} + f\mathbf{k} \times \mathbf{u}_{res} + \frac{\nabla p}{\rho_0} = \frac{1}{\rho_0} \frac{\partial \tau_{eddy}}{\partial z}$$

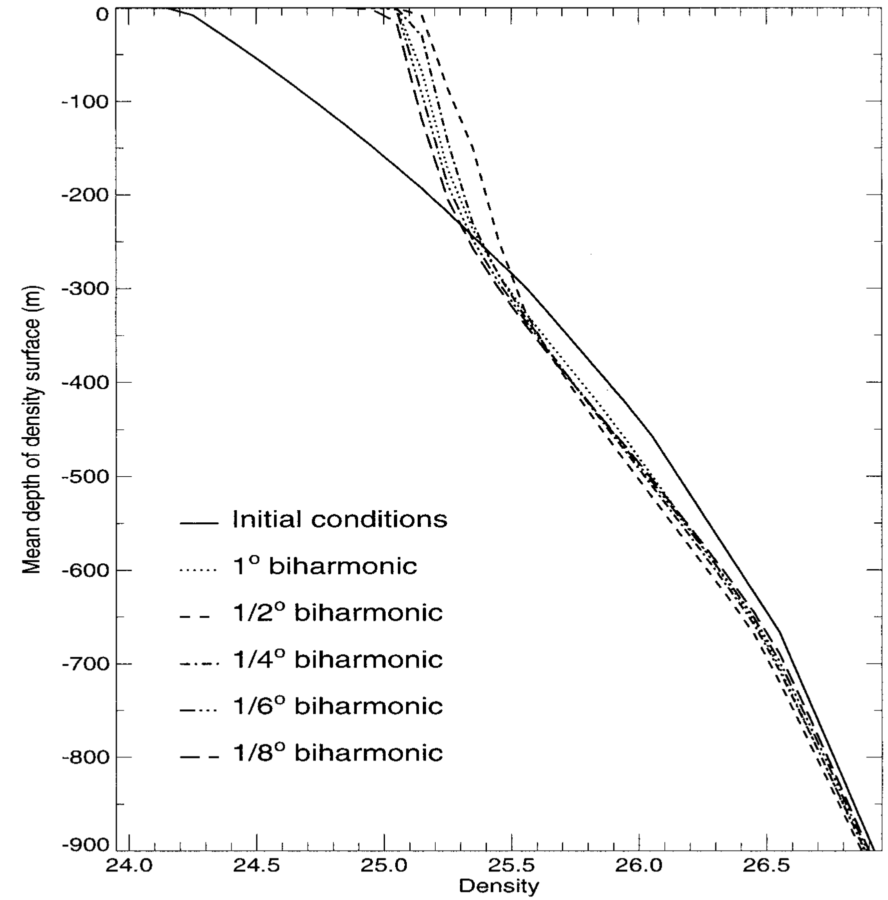
with $\tau_{eddy} = -\rho_0 f \mathbf{k} \times \frac{\overline{b' \mathbf{u}'}}{\overline{b}_z}$

(but still need isopycnal diffusion of tracer)

potential vorticity?

Do we need GM if we partially resolve eddies? (Roberts and Marshall 1998)





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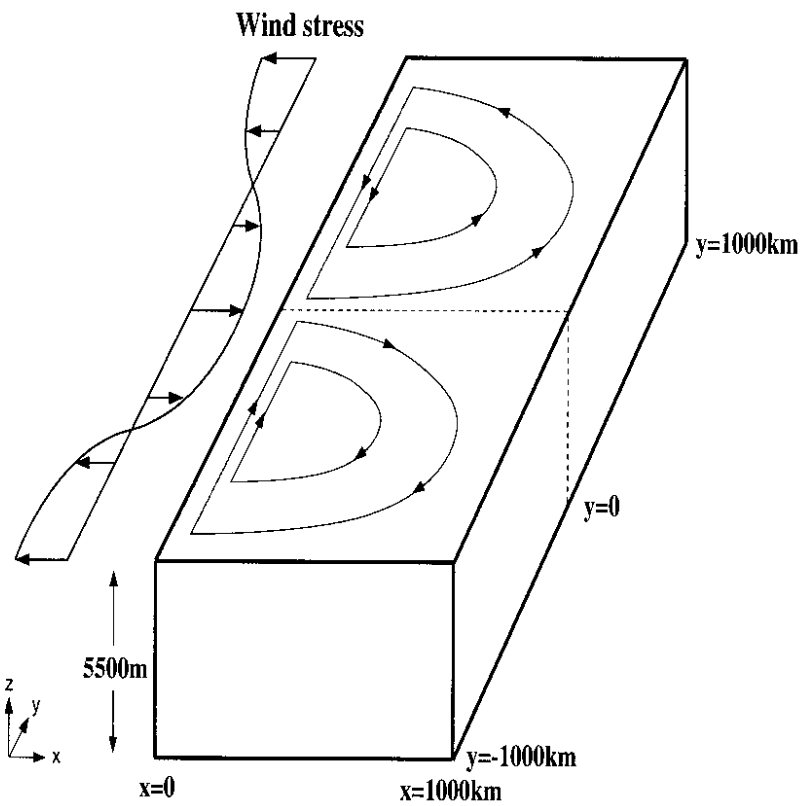
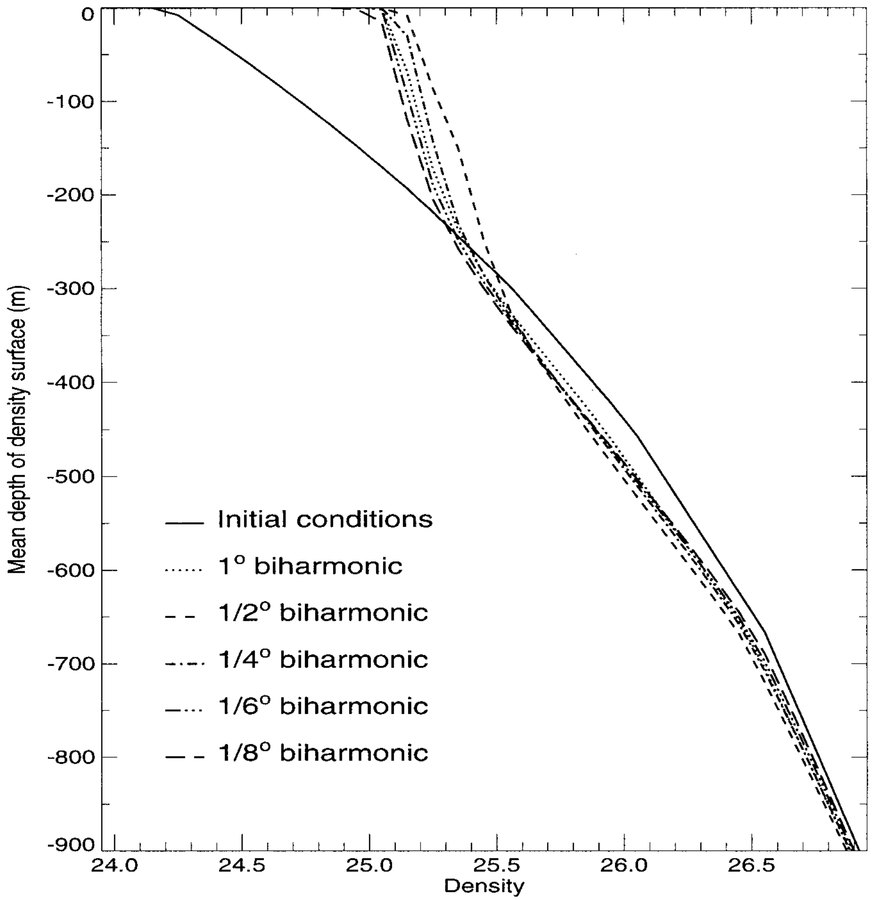
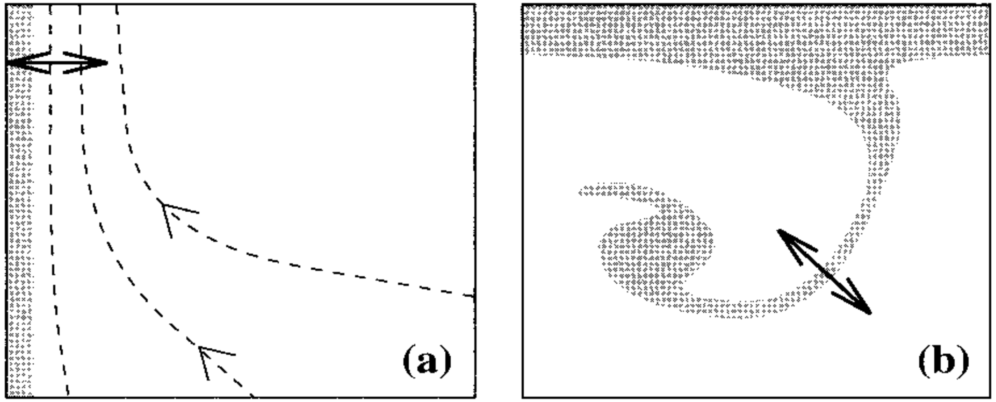




may be due to lateral diffusion implicit in advection schemes (Griffies and Hallberg 2000)

in ICOM, adaptivity will also contribute (very significantly?)

can show that diapycnal transfers are related to vorticity gradients



coarse resolution　　eddy permitting

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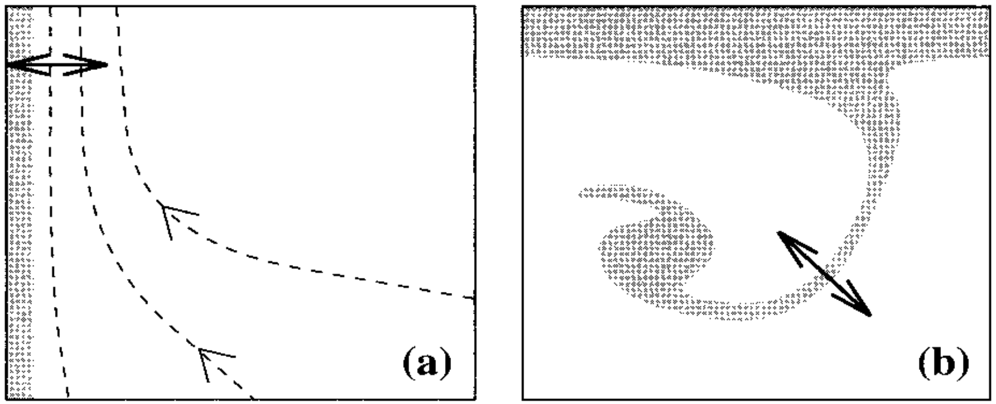

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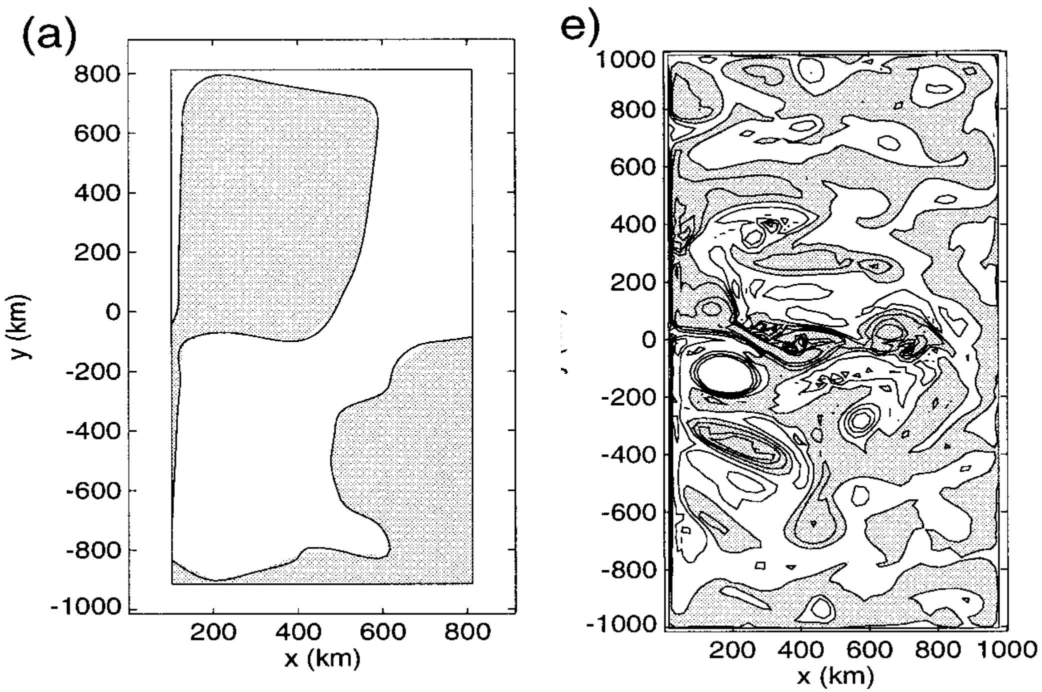

1 degree 1/8 degree

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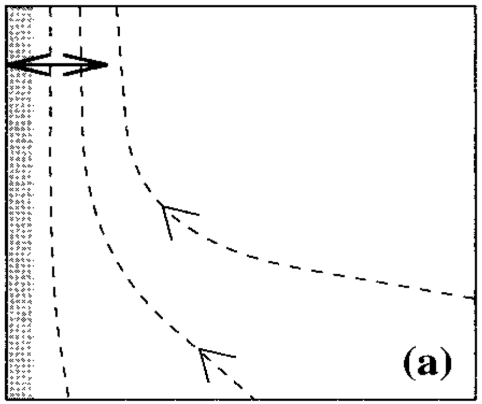

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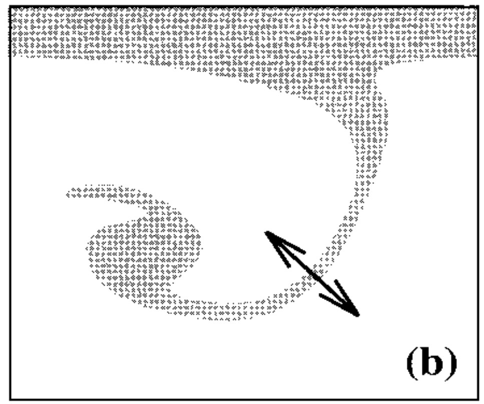

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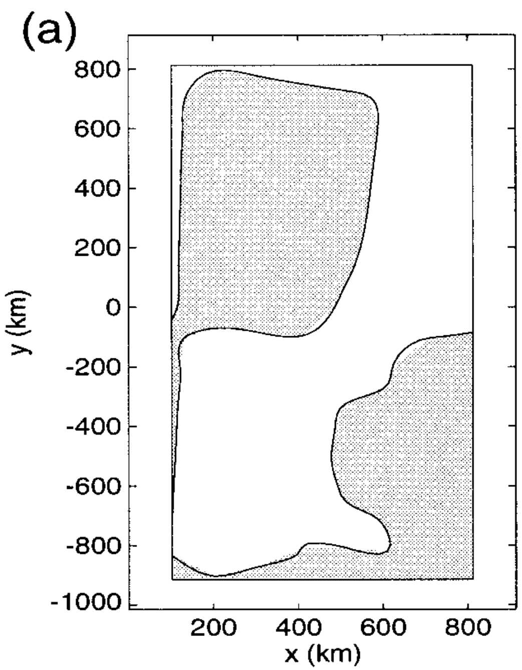

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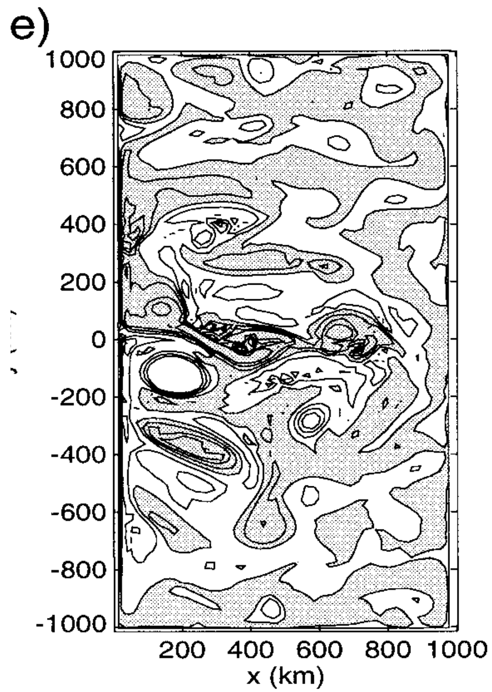

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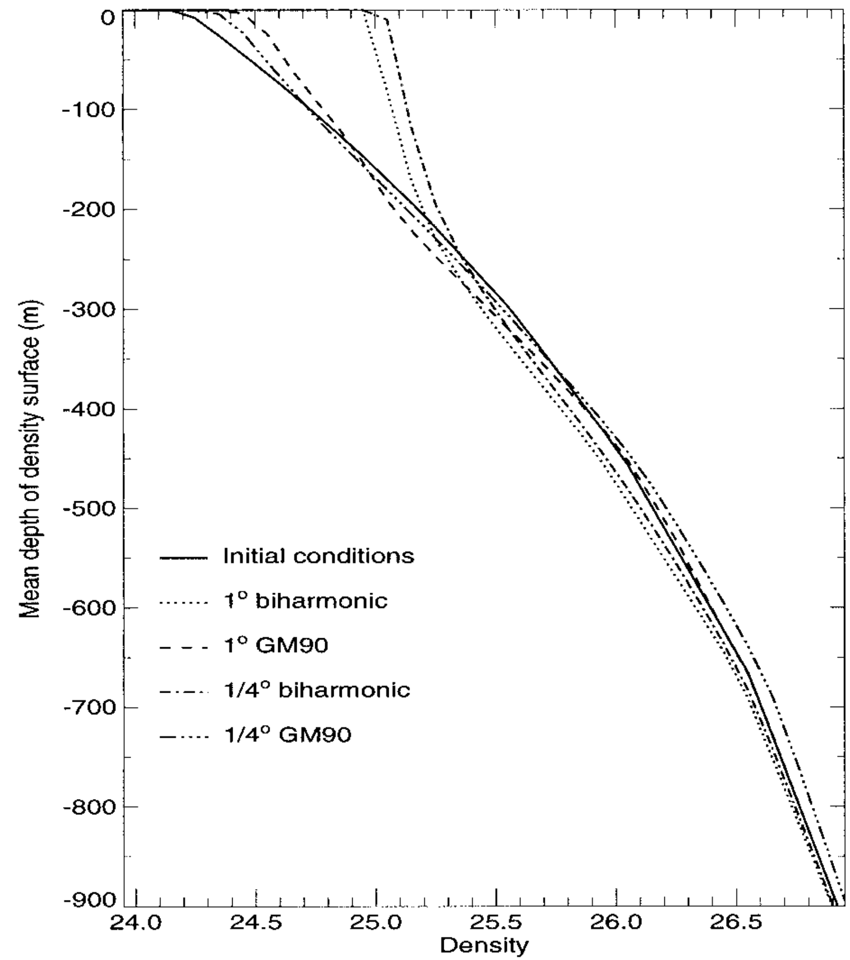
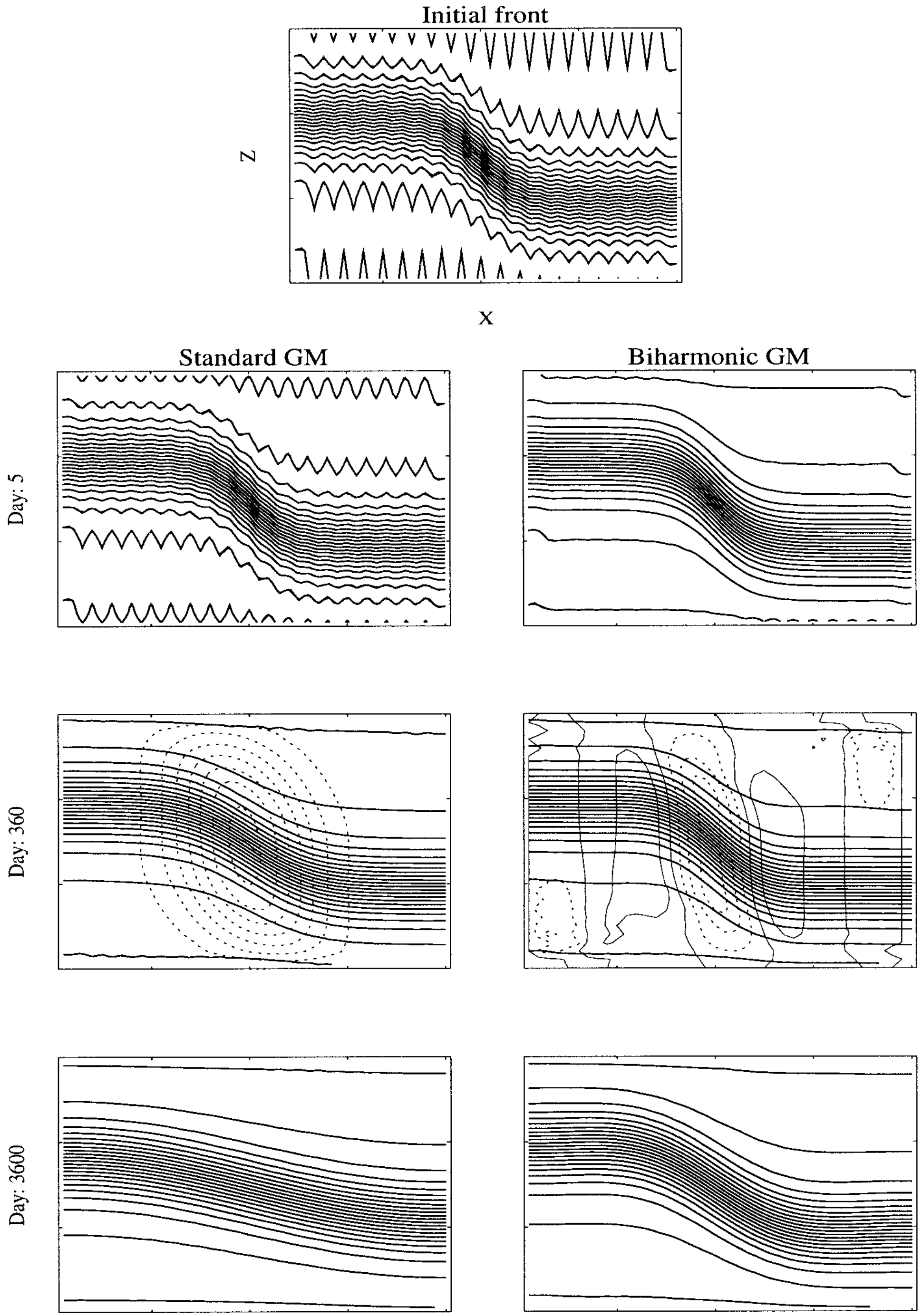
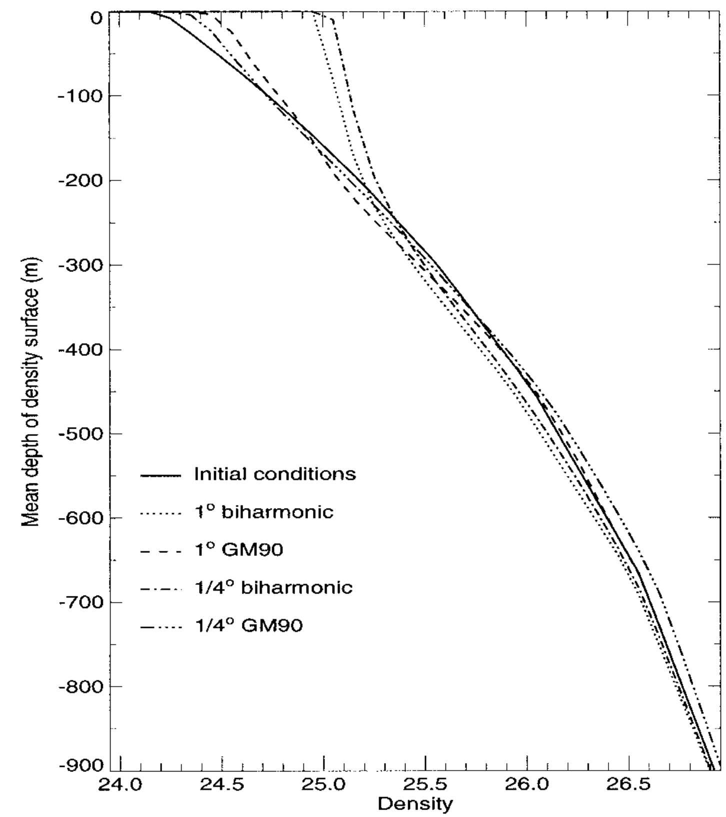


Biharmonic GM

$$\mathbf{u}^* = -\frac{\partial}{\partial z}\left[\boldsymbol{\nabla}\left(\frac{\gamma\nabla^2 b}{\partial b/\partial z}\right)\right] \qquad w^* = \nabla^2\left(\frac{\gamma\nabla^2 b}{\partial b/\partial z}\right)$$





- can also use in conjunction with GM but with smaller eddy transfer coefficient;
- not clear what to do about θ and S!

Summary

- Spurious diapycnal sources/sinks can give large systematic errors in ocean models.
- GM parameterises eddy tracer fluxes as isopycnal diffusion and eddy advection.
- Can implement eddy-induced advection as skew-symmetric diffusion.
- Boundary conditions are an issue (“tapering”) but large literature on this topic.
- Many variations on GM, in particular with different eddy transfer coefficients.
- Still need to arrest the turbulent cascades adiabatically in an eddy-permitting model.